Group Work 10.B.3 Solution

A)

Here's we need to use force balancing about the fulcrum. Summing the torques gives:

$$\sum \tau = m_p g \sin(-90^\circ)(0) + mg \sin(-90^\circ)\left(\frac{-L}{4}\right) + k(x - x_0)\sin(-90^\circ)\left(\frac{L}{2}\right) = I(0)$$

$$0 = mg\left(\frac{L}{4}\right) - k(x - x_0)\left(\frac{L}{2}\right).$$

Thus, we know:

$$mg\left(\frac{L}{4}\right) = k(x - x_0)\left(\frac{L}{2}\right).$$

Multiplying each side by $\frac{4}{gL}$ gives:

$$m = \frac{2k(x - x_0)}{g}.$$

Plugging in values tells us that:

$$m = \frac{(2)\left(200\frac{N}{m}\right)(3m)}{10\frac{m}{s^2}} = 120kg$$

B)

Once again, we begin by summing the torques, and setting α to 0, i.e.:

$$\sum \tau = m_p g \sin(-90^\circ)(0) + mg \sin(-90^\circ) l + k(x - x_0)\sin(-90^\circ)\left(\frac{L}{2}\right) = I(0).$$

Cleaning this up slightly gives:

$$0 = -mgl - k(x - x_0)\left(\frac{L}{2}\right).$$

Adding mgl to each side gives:

$$mgl = -k(x - x_0)\left(\frac{L}{2}\right)$$

$$l = \frac{-k(x - x_0)L}{2mg}.$$

Plugging in values gives:

$$l = \frac{\left(-300\frac{N}{m}\right)(3m)(12m)}{(2)(120kg)\left(10\frac{m}{s^2}\right)} = -4.5m,$$

where the negative sign tells us that the block will be positioned to the left of the block will be to the left of the pivot point.