

Group Work 10.1 Solution

Since the belt is moving along each of the two solid discs without slipping, we can find a relationship between the angular velocity of both discs. By recalling the relationship:

$$v = \omega r.$$

The condition that both discs have the same velocities— $v_{2R} = v_R$ —becomes:

$$2R\omega_{2R} = R\omega_R,$$

which implies us:

$$\omega_R = 2\omega_{2R}. \quad (1)$$

Now, we want to find the rotational energy of each of these discs. Recalling, that rotational kinetic energy is given by the equation:

$$KE_{rot} = \frac{1}{2}I\omega^2,$$

and that for a uniform disc rotating about its center:

$$I = \frac{mr^2}{2},$$

we find that:

$$KE_{2R} = \frac{1}{2} \frac{4mR^2}{2} \omega_{2R}^2 = mR^2 \omega_{2R}^2 \quad (2)$$

and

$$KE_R = \frac{1}{2} \frac{mR^2}{2} \omega_R^2 = \frac{1}{4} mR^2 \omega_R^2. \quad (3)$$

Taking the quotient of equations 2 and 3 gives:

$$\frac{KE_{2R}}{KE_R} = \frac{mR^2 \omega_{2R}^2}{\frac{1}{4} mR^2 \omega_R^2} = 4 \frac{\omega_{2R}^2}{\omega_R^2}. \quad (4)$$

Substituting in equation 1 gives:

$$\frac{KE_{2R}}{KE_R} = 4 \frac{\omega_{2R}^2}{4\omega_{2R}^2} = 1$$