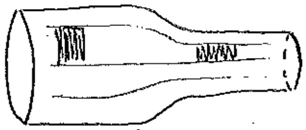
PHYSICS 207: Midterm 3 (2010) solutions

(1)



$\boxed{E}$

(2) The ship will sit at the ~~point when the~~ height when the volume of the ship that is underwater times the density equals the mass of the ship.

Since ship B is narrower at the bottom, ~~the more of it~~ a larger percent of it must sit ~~above~~ below water to achieve the same volume

$\Rightarrow$ B rides lower $\Leftrightarrow$ A rides higher $\Rightarrow$ $\boxed{A}$

(3) The solutions to an equation of the form:

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

is given by:

$$\left(e^{-bt/2m}\right) e^{\pm\sqrt{b^2/4m - k/m}}$$

when the system is ~~over-damped~~ under-damped;

$$\frac{b^2}{4m} - k/m < 0$$

So all damping is determined by:

$$e^{-bt/2m} \Rightarrow \text{only } b, m \text{ affect damping time} \Rightarrow C$$

(4) Fluids are not on test!!!

we know: $A_1 V_1 = A_2 V_2$ (continuity)

$$\tfrac{1}{2}\rho v_1^2 + P_1 = \tfrac{1}{2}\rho v_2^2 + P_2$$

$$\Longrightarrow \tfrac{1}{2}\rho v_1^2 + P_1 = \tfrac{1}{2}\rho\left(\frac{A_1}{A_2}\right)^2 v_1^2 + P_2$$

$$\Longrightarrow \tfrac{1}{2}\rho\left(1 - \frac{r_1^4}{r_2^4}\right)v_1^2 + P_1 = P_2 = \tfrac{1}{2}\rho\left(\frac{r_2^4 - r_1^4}{r_2^4}\right)v_1^2$$

Thus, if $r_1 > r_2$, $P_1 > P_2 \Longrightarrow \cancel{A}, \cancel{B}, \cancel{C}$

It is not linear with respect to $r_2 \Longrightarrow \cancel{E}$

After this it's tricky:

Graph F, looks like the graph of $1/x^n$, but this is $1 - 1/x^n$

$\Longrightarrow$ D

(5.) For a perfect black-body, the power radiated is given by

$P = AT^4$

$\hookrightarrow$ area of the sphere.

There steady state power will be the same in each case

$$\Longrightarrow A_1 T_1^4 = A_2 T_2^4$$

$$= \pi r_1^2 T_1^4 = \pi r_2^2 T_2^4$$

$$\Longrightarrow r_1^2 T_1^4 = (2r_1)^2 T_2^4$$

$$\Longrightarrow \tfrac{1}{4} T_1^4 = T_2^4$$

$$\Longrightarrow T_2 = \frac{T_1}{\sqrt{2}}$$

$$\Longrightarrow \boxed{B}$$

(6) The specific heat of the substance is the amount of heat needed to change a unit mass of the substance by a unit temp, ie.

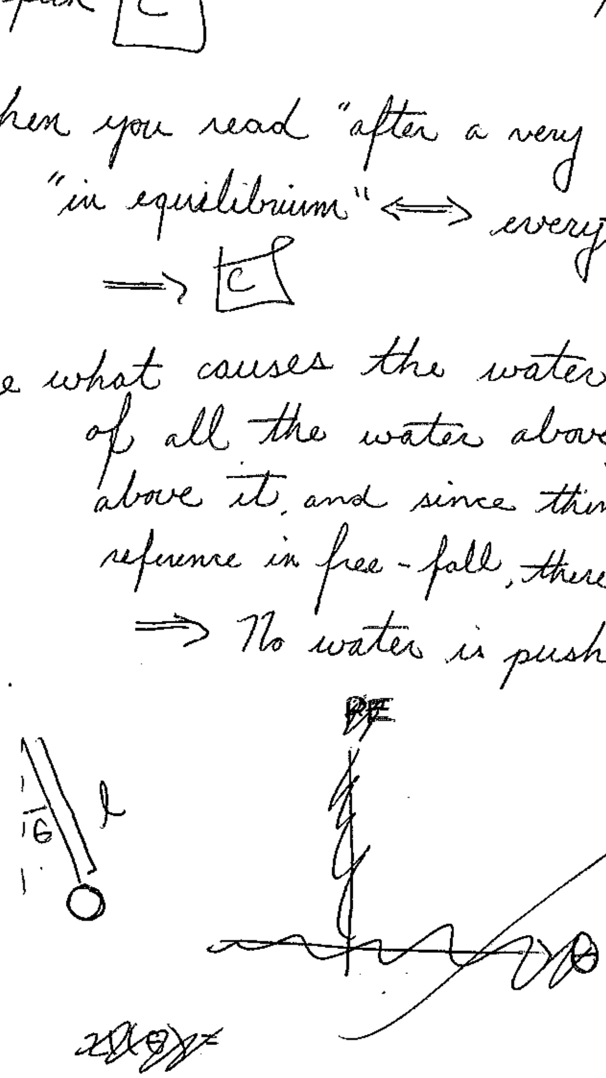
$$\left(\frac{dQ}{dT}\right)\left(\frac{1}{m}\right) = \underbrace{\left(\frac{dQ}{dt}\right)}_{\text{rate at which heat is added}}\left(\frac{dt}{dT}\right)\left(\frac{1}{m}\right) = \left(\frac{dQ}{dt}\right) \Big/ \underbrace{\left(\frac{dT}{dt}\right)}_{\text{slope of the graph}} m$$

Thus, we have something of the form of C, D, and E. Since we want the specific heat of the solid, which is section A, we pick $\boxed{C}$

(7) When you read "after a very long time" you should ~~thing~~ think "in equilibrium" $\Longleftrightarrow$ everything is the same temp.
$\Longrightarrow \boxed{C}$

(8) Since what causes the water to come out was the pressure of all the water above it $\Longleftrightarrow$ the weight of the water above it, and since things are "weightless" in a frame of reference in free-fall, there is no pressure.
$\Longrightarrow$ No water is pushed out $\Longleftrightarrow$ C

(9)



$$\cancel{SHP \Longrightarrow x(t) = A\cos(\omega t + \delta)}$$

$$\cancel{v_{tan}(t) = -A\omega \sin(\omega t - \delta)}$$

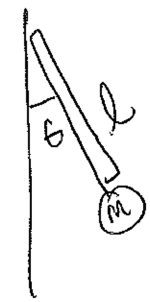
$$\cancel{a_{tan}(t) = -A\omega^2 \cos(\omega t - \delta)}$$

$$\cancel{\Longrightarrow KE = \tfrac{1}{2}A^2\omega^2 \sin^2(\omega t - \delta)}$$

$$\cancel{PE = \tfrac{1}{2}kA^2 \cos^2(\omega t - \delta)}$$

$$\cancel{T = \frac{2\pi}{\omega} = 2\pi\sqrt{m/k}}$$

(9) For a simple harmonic pendulum:



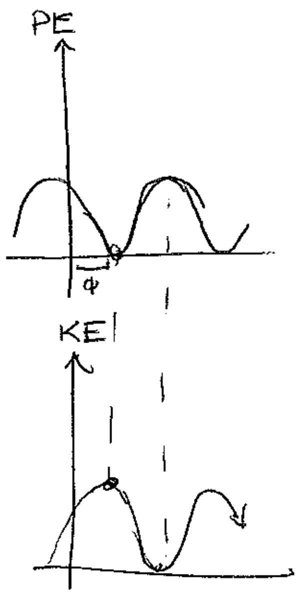

$$\theta(t) = \theta_{max}\sin\left(\sqrt{g/\ell}\,t - \phi\right)$$

$$\frac{d\theta}{dt} = \omega(t) = \frac{v_{tan}}{\ell} = \sqrt{g/L}\,\theta_{max}\cos\left(\sqrt{g/\ell}\,t - \phi\right)$$

$$\frac{d^2\theta}{dt^2} = \alpha(t) = \frac{a_{tan}}{\ell} = -\left(\frac{g}{\ell}\right)\theta_{max}\overset{\sin}{\cancel{\cos}}\left(\sqrt{g/\ell}\,t - \phi\right)$$

$$\Longrightarrow PE \propto \theta_{max}^2\sin^2\left(\sqrt{g/\ell}\,t - \phi\right)$$

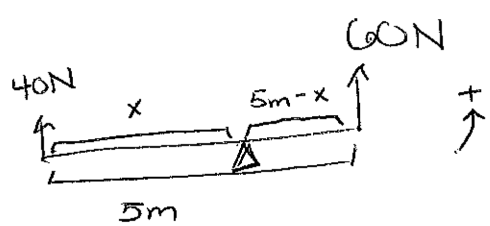
$$KE \propto \theta_{max}^2\left(g/L\right)\cos^2\left(\sqrt{g/\ell}\,t - \phi\right)$$




$T = 2\pi\sqrt{\ell/g} \Longrightarrow$ ~~$\phi$~~ D is wrong

$\boxed{D}$

(10) a)



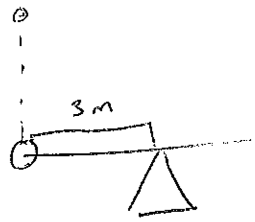

$$\Sigma\tau = (-40N)(x) + (60N)(5m - x) = I(0)$$

$$\Longrightarrow -100N\,x + 300Nm = 0$$

$$\Longrightarrow \cancel{300N}\ +100N(x) = 300Nm$$

$$\Longrightarrow \boxed{x = 3m}$$

(10) (cont.) b) ANGULAR MOMENTUM WILL NOT BE ON THE EXAM ☺☺




$L_i = L_f$

$m_b v_{bi} r_{bi} + I_p \cancel{\omega_i}^{0} = m_b v_{bf} r_b + I_p \omega_f$

$\Longrightarrow m_b r_b (v_i - v_f) = I_p \omega_f$

$(1\,kg)(3\,m)(+10\,m/s - (-4\,m/s)) = (21\,kg\,m^2)\,\omega_f$

$\Longrightarrow \omega_f = +2\ rad/sec$

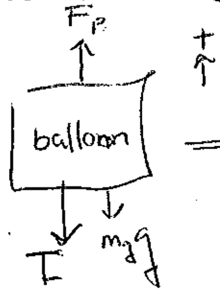
(11) a) $PV = nRT$

$\Longrightarrow n = \frac{PV}{RT}$

$= \frac{(10^5\ Pa)(0.1\,m^3)}{(8.31)(300)}$

$\boxed{= 4.0}$

b) ~~Since the~~ We need to find the tension in the string first.




$\Longrightarrow \sum F = F_B - T - m_g g = m_g (0)$ ↳ since it is moving at constant velo

$\Longrightarrow T = F_B - m_g g$

$= (1000\,kg/m^3)(0.1\,m^3)\,g - (2\,kg)\,g$

$= 980\,N$

(11) b) (cont.) $T = 980N$

$$Y = \frac{T/A}{\Delta L/L} = \frac{(980N/0.05m^3)}{0.0002} = \boxed{9.8E7\ N/m^2}$$

c) $P_{TOT} = P_{atm} + P_{gauge} = P_{atm} + \rho g h$

$= 10^5\ N/m^2 + (1000kg/m^3)(10\ m/s^2)(90m)$

$= 10^6\ N/m^2$

$P_1 V_1 = P_2 V_2$

$\Longrightarrow V_2 = \frac{P_1}{P_2} V_1$

$= \left(\frac{10^5}{10^6}\right) 0.1m^3$

$= 0.01\ m^3$

Since the velocity is still constant

$\Sigma F_y = F_B - T - m_g g = m_g (0)$

$\Longrightarrow T = F_B - m_g g$

$= (1000\ kg/m^3)(0.01m^3) g - 2kg\, g$

$\boxed{= 80N}$

(12) $Q = L_f m = L_f \rho_{ice}(\ell A)$

↳ Latent heat of fusion is heat needed per unit mass

$$\underbrace{\frac{Q}{t}}_{\text{how much heat makes it through the ice per time}} = (K_{ice})\left(\frac{A}{L}\right)\Delta T$$

$L$ ↳ thickness of ice

$\Delta T$ ↳ temp difference between the two sides of the ice

$$\Longrightarrow \frac{L_f \rho_{ice}(\ell A)}{t} = K_{ice}\left(\frac{A}{L}\right)\Delta T$$

$$\Longrightarrow \ell = (K_{ice})(\Delta T)\left(\frac{t}{\rho_{ice} L \cdot L_f}\right)$$

$$\boxed{= 0.0026\,m}$$

(13) a) $\omega = \sqrt{k/m}$

$$= \sqrt{\frac{200\,N/m}{2.0\,kg}} = \sqrt{100}\ rad/sec = \boxed{10\ rad/sec}$$

$2.0\,kg$ ↳ mass of scale + block

b) $y(t) = y_0 \cos(\omega t - \phi)$

since it starts at rest, $\phi = 0$

$y(t) = y_0 \cos(\omega t)$

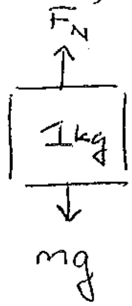
$v(t) = -\omega y_0 \sin(\omega t)$

$a(t) = -\omega^2 y_0 \cos(\omega t)$

$= -(10\,Hz)^2(-0.2\,m)$

$$\boxed{= 20\,m/s^2}$$

(13) (cont.) c) $T = \frac{2\pi}{\omega} \Longrightarrow \frac{T}{4} = \frac{\pi}{2\omega}$; $a = -\omega^2 y_0 \cos(\omega(\pi/2\omega)) = 0$




$$\sum F_y = F_N - mg = m(0)$$

$$\Longrightarrow F_N = mg$$

$$= (1\,kg)(10\,m/s^2)$$

$$\boxed{= 10N}$$

~~d) We want to know if $F_N = 0$~~ the acceleration is such ~~that $F_N = 0$, ie is the scale~~ ~~ever~~

d) We want to know if the scale is every accerating away from the block faster than gravity is pulling it down. i.e. math, does

$-10\,m/s^2 = a(t)$ have a solution

$$-10\,m/s^2 = -\omega^2 y_0 \cos(\omega t)$$

$$= (100/s^2)(0.2\,m)\cos(\omega t)$$

$$\Longrightarrow \cos(\omega t) = -0.5$$

So this has a solution at

$$\omega t = 2.09$$

$$\Longrightarrow t = 2.09/10\,Hz = \boxed{0.209\ sec}$$

(14) a) $\frac{PV}{T} = \cancel{\text{const}}$ $\quad PV = nRT$

$$\Longrightarrow \frac{PV}{T} = nR$$

Thus T is at max when PV is at max, and min when PV is min.

$$T_A = \frac{P_A V_A}{nR}$$

$$= \frac{(16600\,N/m^3)(0.5\,m^3)}{(2\,mol)(8.314)}$$

$$\approx 500K$$

$$T_D = \frac{(8300\,N/m)(0.5\,m^3)}{(2\,mol)(8.314)}$$

$$= 250\,K$$

$$\Longrightarrow \Delta T = \cancel{500}\ 250K$$

b) ~~B → C:~~ $\cancel{Q = nC_v R\Delta T}$
~~(constant volume)~~ (constant pressure)

$$\cancel{= (2)(5/2)(8.3)(-250)}$$

$$\cancel{= -10{,}375\,J}$$

$$\cancel{W_{on\ sys} = PdV} \quad \cancel{\frac{P_A V_A}{T_A} = \frac{P_B V_B}{T_B} \Longrightarrow \left(\frac{T_B}{T_A}\right)\left(\frac{V_A}{V_B}\right)P_A = P_B}$$

$$\cancel{P_B = \left(\frac{250k}{500k}\right)\left(\frac{0.5\,m^3}{2.0\,m^3}\right)16600\,N/m =}$$

$$\cancel{= 2075\,N/m}$$

★ Don't forget to actually make table ★

(14) (cont.) b) $B \longrightarrow C$: $Q = nC_P R\Delta T$

(cons. pressure)

$= (2)(5/2)(8.3)(250)$

$= 10{,}350\,J$

$P_A V_A = P_B V_B \Longrightarrow P_B = P_A \left(V_A / V_B\right)$

$= (16600\,N/m)\left(\frac{0.5\,m^3}{2.0\,m^3}\right)$

$= 4150\,N/m$

$W_{\text{done on sys}} = -PdV = -P\Delta V$

↳ this equality only hold if P is cons.

$= (-4150\,N/m)(1\,m^3 - 2\,m^3) = 4150\,J$

$\Delta E_{TH} = Q + W_{DoS} = -6200\,J$

$C \longrightarrow D$: $\Delta E_{TH} = 0$; $W_{DoS} = -PdV = -\frac{nRT}{V}dV = -nRT \ln\left(V_f / V_i\right)$

(isothermal)

$= (-2)(8.3)(250)\ln(0.5/1)$

$= 2880\,J$

$\Longrightarrow Q = -2880\,J$

$D \longrightarrow A$: $Q = nC_V R\Delta T$

(cons volume)

$= (2)(3/2)(8.3)(250)$

$= 6200\,J$

$W_{DoS} = PdV = 0$ [since V cons]

$\Longrightarrow \Delta E_{TH} = 6200\,J$